

3. (i)

D(14) Maths
paper-1st, Gr-A
Summation of
Series by crs method

$$C = 1 + \frac{\cos \theta}{L1} + \frac{\cos 2\theta}{L2} + \frac{\cos 3\theta}{L3} + \dots \text{to } \infty$$

$$S = \frac{\sin \theta}{L1} + \frac{\sin 2\theta}{L2} + \frac{\sin 3\theta}{L3} + \dots \text{to } \infty$$

$$\begin{aligned}
 \therefore CHS &= 1 + \frac{1}{L} (\cos\theta + i\sin\theta) + \frac{1}{L^2} (\cos 2\theta + i\sin 2\theta) + \frac{1}{L^3} (\cos 3\theta + i\sin 3\theta) + \dots \\
 &= 1 + \frac{e^{i\theta}}{L} + \frac{1}{L^2} e^{i2\theta} + \frac{1}{L^3} e^{i3\theta} + \dots
 \end{aligned}$$

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$$\begin{aligned}
 &= 1 + \frac{x}{L} + \frac{x^2}{L^2} + \frac{x^3}{L^3} + \dots \\
 &= e^x = e^{\frac{e^{i\theta}}{L}} = e^{\cos\theta + i\sin\theta} \\
 &= e^{\cos\theta} e^{i\sin\theta}
 \end{aligned}$$

$$\begin{aligned}
 CHS &= e^{\cos\theta} \{ \cos(\sin\theta) + i\sin(\sin\theta) \} \\
 \therefore C &= e^{\cos\theta} \cos(\sin\theta) \\
 S &= e^{\cos\theta} \sin(\sin\theta)
 \end{aligned}$$

4(iii)

$$S = \sin\theta - \frac{\sin 2\theta}{L^2} + \frac{\sin 3\theta}{L^3} - \dots$$

$$C = \cos\theta - \frac{\cos 2\theta}{L^2} + \frac{\cos 3\theta}{L^3} - \dots$$

$$\begin{aligned}
 \therefore CHS &= (\cos\theta + i\sin\theta) - \frac{1}{L^2} (\cos 2\theta + i\sin 2\theta) + \frac{1}{L^3} (\cos 3\theta + i\sin 3\theta) - \dots \\
 &= e^{i\theta} - \frac{e^{2i\theta}}{L^2} + \frac{e^{3i\theta}}{L^3} - \dots
 \end{aligned}$$

1111 $e^{i\theta} = x$

$$\therefore CHS = x - \frac{x^2}{L^2} + \frac{x^3}{L^3} - \dots$$

$$\begin{aligned}
 &= 1 - e^{-x} = 1 - e^{-\frac{e^{i\theta}}{L}} = 1 - e^{-(\cos\theta + i\sin\theta)} \\
 &= 1 - e^{-\cos\theta - i\sin\theta} \\
 &= 1 - e^{-\cos\theta} \{ \cos(\sin\theta) - i\sin(\sin\theta) \} \\
 &= 1 - e^{-\cos\theta} \cos(\sin\theta) + i e^{-\cos\theta} \sin(\sin\theta) \\
 \therefore C &= 1 - e^{-\cos\theta} \cos(\sin\theta)
 \end{aligned}$$

$$S = e^{-\cos \theta} \sin(\sin \theta) \quad \text{Ans}$$

(2)

Rough: $x - \frac{xL}{L} + \frac{x\beta}{L} - \frac{x\gamma}{L} + \dots$

$$1 - 1 + \frac{xL}{L} - \frac{x\beta}{L} + \frac{x\gamma}{L} + \dots$$

$$= 1 - \left\{ 1 - \frac{xL}{L} + \frac{x\beta}{L} - \frac{x\gamma}{L} + \dots \right\}$$

$$= 1 - e^{-x}$$

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S. III $C = 1 + u \cos x + u^2 \cos^2 x + \dots$ to ∞

$S = x \sin x + u^2 \sin^2 x + \dots$ to ∞

CHB $= 1 + u(\cos^2 + i \sin^2) + u^2(\cos^2 + i \sin^2) + \dots$

$u e^{i(x+y)} = 1 + u e^{ix} + u^2 e^{2ix} + u^3 e^{3ix} + \dots$

$$= 1 + y + y^2 + y^3 + \dots = \frac{1}{1-y} = \frac{1}{1-u e^{ix}}$$

$$= \frac{1}{1-u(\cos^2 + i \sin^2)} = \frac{1}{(1-u \cos^2) - i u \sin^2}$$

$$= \frac{(1-u \cos^2) + i u \sin^2}{(1-u \cos^2)^2 + u^2 \sin^2}$$

$$= \frac{(1-u \cos^2) + i u \sin^2}{1 - 2u \cos^2 + u^2 \cos^2 + u^2 \sin^2}$$

$$= \frac{(1-u \cos^2) + i u \sin^2}{1 - 2u \cos^2 + u^2 \cos^2 + u^2 \sin^2}$$

$$1 - 2u \cos^2 + u^2 \cos^2 + u^2 \sin^2$$

Equating real parts.

$$C = \frac{1 - u \cos^2}{1 - 2u \cos^2 + u^2 \cos^2 + u^2 \sin^2}$$

$$= \frac{1 - u \cos^2}{1 - 2u \cos^2 + u^2}$$

$$= \frac{1 - u \cos^2}{1 - 2u \cos^2 + u^2} \quad \underline{\quad}$$