3. (vi)

D.I.(H) Maths paper-1st, Gr-A

summation of series by C+iS method

$$C = 1 + \frac{\cos\theta}{\lfloor 1} + \frac{\cos 2\theta}{\lfloor 2} + \frac{\cos 3\theta}{\lfloor 3} + \cdots \text{ to } \infty$$

and

$$S = \frac{\sin\theta}{\lfloor 1} + \frac{\sin 2\theta}{\lfloor 2} + \frac{\sin 3\theta}{\lfloor 3} + \cdots \text{ to } \infty$$

(1)

$\therefore C+iS = 1+\frac{1}{\lfloor 1}(\cos\theta+i\sin\theta)+\frac{1}{\lfloor 2}(\cos2\theta+i\sin2\theta)+\frac{1}{\lfloor 3}(\cos3\theta+i\sin3\theta)$

$= 1+\frac{e^{i\theta}}{\lfloor 1}+\frac{1}{\lfloor 2}e^{i2\theta}+\frac{1}{\lfloor 3}e^{i3\theta}+\cdots$

$e^{i\theta}=x$

$= 1+\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\cdots$

$= e^{x} = e^{e^{i\theta}} = e^{(\cos\theta+i\sin\theta)}$

$= e^{\cos\theta}e^{i\sin\theta}$

$C+iS = e^{\cos\theta}\{\cos(\sin\theta)+i\sin(\sin\theta)\}$

$\therefore C = e^{\cos\theta}\cos(\sin\theta)$

$S = e^{\cos\theta}\sin(\sin\theta)$ Ans

4(iii) $S = \sin\theta-\frac{\sin2\theta}{\lfloor 2}+\frac{\sin3\theta}{\lfloor 3}-\cdots\infty$

$C = \cos\theta-\frac{\cos2\theta}{\lfloor 2}+\frac{\cos3\theta}{\lfloor 3}-\cdots$

$\therefore C+iS = (\cos\theta+i\sin\theta)-\frac{1}{\lfloor 2}(\cos2\theta+i\sin2\theta)+\frac{1}{\lfloor 3}(\cos3\theta+i\sin3\theta)$

$= e^{i\theta}-\frac{e^{2i\theta}}{\lfloor 2}+\frac{e^{3i\theta}}{\lfloor 3}-\cdots$

माना $e^{i\theta}=x$

$\therefore C+iS = x-\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}-\cdots$

$= 1-e^{-x} = 1-e^{-e^{i\theta}} = 1-e^{-(\cos\theta+i\sin\theta)}$

$= 1-e^{-\cos\theta}\cdot e^{-i\sin\theta}$

$= 1-e^{-\cos\theta}\{\cos(\sin\theta)-i\sin(\sin\theta)\}$

$= 1-e^{-\cos\theta}\cos(\sin\theta)+ie^{-\cos\theta}\sin(\sin\theta)$

$\therefore C = 1-e^{-\cos\theta}\cos(\sin\theta)$

$S = e^{-\cos\theta}\sin(\sin\theta)$ Ans

Rough: $x - \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} - \frac{x^4}{\lfloor 4} + \cdots$

$1 - 1 + x - \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} - \frac{x^4}{\lfloor 4} + \cdots$

$= 1 - \left\{1 - x + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \cdots\right\}$

$= 1 - e^{-x}$

0081.

5. III $C = 1 + x\cos\alpha + x^2\cos 2\alpha + \cdots$ to ∞

$S = x\sin\alpha + x^2\sin 2\alpha + \cdots$ to ∞

$C + iS = 1 + x(\cos\alpha + i\sin\alpha) + x^2(\cos 2\alpha + i\sin 2\alpha) + \cdots$

$xe^{i\alpha} = y$ (let) $= 1 + xe^{i\alpha} + x^2e^{2i\alpha} + x^3e^{3i\alpha} + \cdots$

$= 1 + y + y^2 + y^3 + \cdots = \frac{1}{1-y} = \frac{1}{1 - xe^{i\alpha}}$

$= \frac{1}{1 - x(\cos\alpha + i\sin\alpha)} = \frac{1}{(1 - x\cos\alpha) - ix\sin\alpha}$

$= \frac{(1 - x\cos\alpha) + ix\sin\alpha}{(1 - x\cos\alpha)^2 + x^2\sin^2\alpha}$

$= \frac{(1 - x\cos\alpha) + ix\sin\alpha}{1 - 2x\cos\alpha + x^2\cos^2\alpha + x^2\sin^2\alpha}$

Equating real parts.

$C = \frac{1 - x\cos\alpha}{1 - 2x\cos\alpha + x^2(\cos^2\alpha + \sin^2\alpha)}$

$= \frac{1 - x\cos\alpha}{1 - 2x\cos\alpha + x^2}$ Pr